Logistics Management for Improving Service Level of Toll Plaza by Mathematical Model and Simulation

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Abstract

Many urban areas and states are facing an increase in population density and the industrial base, creating traffic congestion and jams at toll plazas on major highways during rush hours. This article provides a detailed study and analysis of the performance evaluation of a toll plaza using mathematical model with queuing theory and discrete event simulation modeling with ARENA. Under a given time-dependent traffic flow and a set of specified model parameters, the required optimal number of tollbooths and the resulting mean delay can be estimated with the discrete event simulation model. The results show that a toll plaza has a shorter delay under light load conditions is more sensitive of variations in service type and the number of toll plazas. And show the performance of toll plazas improves as the service process payment at tollbooths has mix and flexible to used by customers because less time is spent in queue.

Keywords: Mathematical model, model and simulation, toll plaza, electronic toll collection, service planning

1. Introduction

The traffic jams usually has importance problem around the world, the good operating and management that only way to solving the traffic problem to smoothly and efficiency. In the normal traffic to looking around, many driver does not going faster to destination why the express way and highway has constructed to response the traveler go to destination in shorting time and reduce density of traffic to quickly. In Thailand have Express Way to service driver and organizing by Expressway Authority of Thailand.

In contrast, dynamic lane configuration could be used as a preventive measure in anticipation of non-stationary traffic volume in each lane type over time in order to be integrated with the work force scheduling. Another important issue related to toll plazas is capacity. Although dynamic lane configuration policy may be optimally employed in current facilities, toll users may still suffer from long wait times if the capacity is not enough to handle all vehicles, especially during rush hours. If new toll plazas are to be designed, we should plan for appropriate capacities not only for the current traffic volume but also for increases in future traffic volume. Some toll plazas may have enough capacity now, but sometime in the future its capacity may not be sufficient to handle all vehicles during peak times due to increased traffic volume. Since it is not possible to continuously expand the capacity of toll plazas due to geographical restrictions, it may be advisable to devise a strategy to encourage motorists currently using low-throughput lanes, such as manual lanes, to move gradually to high-throughput lanes like electronic toll collection lanes (hereafter referred to as ETC lanes). Although this may be a long-term strategy, if successful, it may have a greater impact in resolving congestion problems at toll plazas.

Motivated from the above problems, this research focuses on mainly issues: (1) design of a toll plaza with the right initial capacity, (2) operations with dynamic lane configuration over time, and (3) evaluation of a long-term strategy to transfer motorists from low-throughput to high-throughput lanes (4) estimation of performance at toll plaza from current situation and the planning design by queuing math model (Hiller and Lieberman et al, 2005; Winston, 1994) and simulation with ARENA version 10 (Kelton et al, 2007) (5) comparison of methods of queuing model and simulation for crosscheck the solution. We evaluate how effective this strategy is in reducing long waits in the low-throughput lanes and, thus, improving overall performance at toll plazas.

2. The applied from queuing theory to the toll plaza

Waiting queue system or waiting time is one of the parts of daily life, i.e. from the morning, if they go to work or school by bus, sky train, or subway, it would first to meet queue for going in public vehicle or train (in actually, they not see the queue has well-disciplinary) (Vipawaan, 2545). From the number of time loss, the population in countries has been call one factor which affect to quality of life and effective of economics in many countries i.e. in USA has estimate total time of all population would be lose from the waiting time to 37,000,000,000 hour per year. If they used time to construction job that would be hire in 20 million people per year (Hiller and Lieberman, 2005). From the any waiting time problem that has been made the queuing theory by used logical of mathematical in order to analyzed and find the way to solve in each problem, the result should help to reduce cost or management in service system.

Many studies have focused on operating a toll plaza based on only the current traffic volume. However, as far as we know, the research study of Kim (2009) is the first research to provide comprehensive decision-making procedures for designing a new toll plaza with the right initial capacity, then determining optimal dynamic lane configuration when operating the toll plaza over time, evaluating the effects of increasing future traffic volume on waiting time by sensitivity analysis, and establishing a long-term quantitative strategy by transferring low-throughput lane users to high-throughput lanes in anticipation of increases in future traffic volume. The final contribution is to illustrate the application of the introduced methods to a case study based on actual traffic data collected.

Optimization problems for toll plaza operations are related to queuing studies because arrival and service processes are not deterministic but stochastic in nature. Initial work on optimal control policies for M/G/1 queuing systems can be found in Heyman (1968) and its extensions with balking and reneging are presented in Blackburn (1972) and Mendelson (1985). Queuing systems with two servers are analyzed in Bell (1980). Szarkowicz (1985) examines general cases of queuing systems with more than two servers. Toll plaza operation is also closely related to pricing issues. I.e. the case of Klodzinski and Al-Deek (2004) reported on the basis of real data that the mean service rates per hour for ETC (Electronic Toll Collection), ACM (Automatic Coin Machine), and manual lanes were 1708, 503, and 376, respectively. Although these numbers are toll plaza specific, it is generally understood that the mean service rate for the ETC lane is the highest and the one for the manual lane is the lowest. The variance for the ETC lane is also the smallest and the one for the manual lane is the largest.

Especially, the case of Kim (2009) is so interest because the system to analyze data and the type of vehicle to serve service has been approach in this paper. Kim (2009) has analyzed average arrival rate in each time and each type of payment in figure 1. Let denote the symbol λ mean the arrival rate in all vehicle type, and classified to λ_E mean to arrival rate of customer used payment by ETC system, λ_M mean arrival rate of customer used payment by ACM system, and λ_M mean arrival rate of customer used payment by manual system.

The case of Kim (2009) determines the appropriate capacity of the new toll plaza. Values of bi are computed for each payment type over time in figure 3 so that the queuing system reaches a stable steady-state condition with finite queues. We have $b_E + b_A + b_M = 1 + 3 + 8 = 12$ between 7:00 and 8:00 when the traffic volume is highest during a day. Thus, if its capacity is set to at least 12 toll booths, the mean arrival rate becomes less than the mean service rate for each lane, and the new toll plaza will remain stable with finite queues. Although the minimum required number of lanes for a stable queuing system is identified as 12, toll plaza officials set the capacity equal to 16, the maximum possible number under the geographical restrictions, due to possible increases in traffic volume in the future.

3. The complex of toll collection plaza applied by simulation with arena

Highways and freeways in urban areas are experiencing severe traffic congestion and jams due to the increasing population and migration of people to the cities and urban areas. This issue is of great concern to city and urban planners and engineers, as well as to transportation engineers and planners. It is important to build high-quality freeways that can be maintained on a regular basis using good methods. It is also important to provide enough funds to do so, and using the toll concept is one way to have enough funds to maintain such highways. Good designs of these toll plaza systems can have a significant impact on the effective use of the infrastructure and can contribute to increasing the standards and quality of living of the residents in urban areas (van Dijk et al, 1999; Matstoms, 1995; Pursula, 1999; Chao, 1999; Edie, 1954; Burris and Hildebrand, 1996; Belenky, 1998; Sadoun, 2001; Sadoun, 2003; Sadoun, 2003, Sadoun, 2005).

In 1954, Edie (1954) conducted a study in which he considered traffic delays at tollbooths of homogeneous booths and vehicles. When we have different types of traffic, the problem becomes more challenging as the analysis becomes much more difficult. Among the possible techniques that can be used to analyze the performance of all transportation systems, including tollbooth plazas of highways, are operation research techniques (Sadoun, 2001). van Dijk et al. (1999) presented different queuing models for the simulation of the toll plaza. Among the variety of commonly used queuing configurations under different scenarios are the ones illustrated in Figure 3.

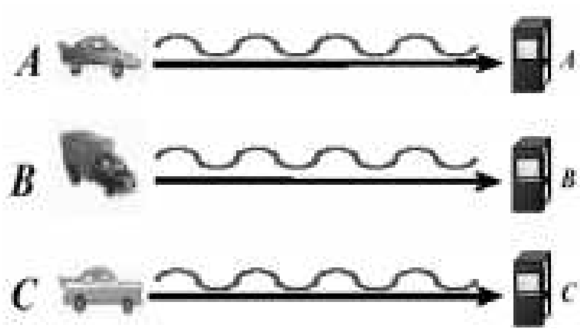

Figure 3: Queuing system 1: Separate line (van Dijk et al, 1999)

One option is to separate the lanes and to offer only one type of payment system at every tollbooth (Fig. 3). The disadvantage of a separate-lane system is that certain tollbooths might be underused while others would be overloaded. This can be inefficient in operational usage. Analytical queuing systems can predict the behavior of separate-lane queuing systems well

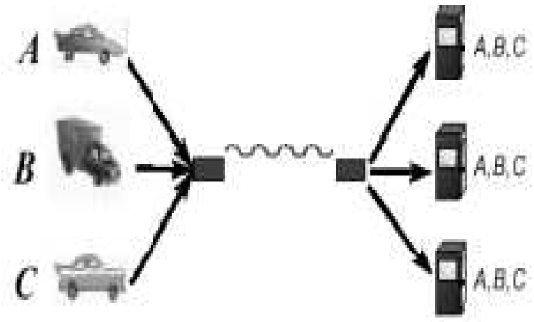

Figure 4: Queuing system 2: One line (van Dijk et al, 1999)

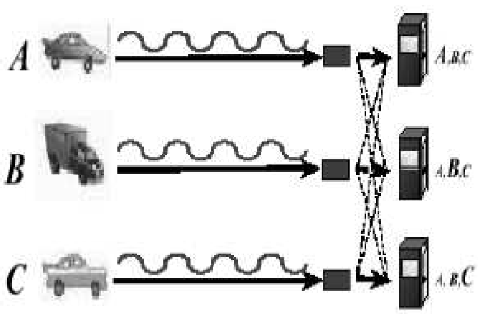

Figure 5: Queuing system 3: specification and overflow (van Dijk et al, 1999)

Another option would be to offer all payment systems at all tollbooths and have the vehicles queue in one line (see Fig. 4). Each toll booth would accept cash, credit cards, and electronic payment, or it would be a homogeneous-type toll plaza. The advantage in this case is that it provides more operational flexibility while also increasing the efficiency (utilization) of the tollbooths. The disadvantage, however, is that the wide range of service times introduces variability. As variability in service times is one of the main causes for queues to arise, this disadvantage could override the gain in efficiency. In fact, by combining both insights, a third option might even be more attractive by using specialized lanes to keep the variability per lane to a minimum and by also allowing "overflow" when one or more ticket booths are temporarily underused (see Fig. 5). Among all three options, the more efficient option 2 (one-line system) has an overall better performance (van Dijk et al. 1999).

The applied of waiting queue toll plaza with simulation has studied by Zarrillo et al. (1998). The model of waiting time at the electronic toll plaza in the rush hour and studied many factors affect to the capacity of toll plaza. Fambro and Rouphail (1997) has create the simulation model the waiting queue at the cross road signal to applied in ETC and have constrain with the development system of time why reduce speed and increase speed has not record. Sisson (1995) has considered the benefit from decrease of air pollution. Saka et al (2001) used the simulation for make the operation traffic model and measured affect of electronic toll collection machines, from the literature studies of simulation model has represent the ETC can help to decrease traffic volume, reduce the traveling time, the traffic has driving to be smoothly, and decrease the air pollution, the result is more advantage in term of economics and environmental. The advantage has been increase if the expressways have used this machine in all system.

4. The research Methodology

In the case of this paper has used the queuing theory to analyze by M/M/s system (Winston, 1994; Hiller and Lieberman et al, 2005). for consider the system of service at toll plaza in any type of planning form, if they change the behavior of process toll collection that how affect to the service system. The distribution of service time is character in exponential distribution; the service station has more than one.

This paper has study how to development the toll collection system in 3 characters as follow:

1. Assignment a specific lane to a specific type of payment at toll collection system
2. Assignment some specific lane to a specific type of payment while the other can be mixed excepted the ETC lane system at toll collection system
3. Assignment all specific lane to all specific type of payment has be mixed

The data has used case study from the observation and statistic of the customers used the expressway at the dao khanong toll collection plaza (see figure 6). The infrastructure of the dao khanong toll plaza had the specific lane in 9 lanes and the capacity of vehicle to service in 45,129 trip/day. The many problems from the traffic jam in front of toll collection plaza and the decision to how many gate open the toll booths, from the calculation math model can help the result to decision support to consider the number of booths has open at the toll plaza.

Figure 6: the location of toll collect plaza at Dao Khanong station (ref. from Google map)

This paper has assigned the assumption for considered the queuing system by the queuing theory as:

- The type of collection system at the toll plaza in 4 characteristics as:
  - The customers has payment the money by cash changes
  - The customers has payment the money by cash without changes
  - The customers has payment the money by coupon
  - The customers has payment the money by electronic pass
- The character of arrival has follow from First Come First Serve (FCFS)
- The number of toll collection booths has total 9 lane, assume the character of the same type of the toll collection system has stand in near in order to easy to analyzed and this paper has decide the character in the detail as follow from table 1:

Table 1: The definition time process of toll plaza using in queuing theory

	Process Type	**Customers Arrival**	**Service Time**	**Server**
case 1	Payment by cash	15 trips/minute	7.5 trips/minute/lane	3
	Payment by cash without changes	10 trips/minute	12 trips/minute/lane	2
	Payment by coupon	3 trips/minute	15 trips/minute/lane	2
	Payment by ETC	4 trips/minute	15 trips/minute/lane	2
case 2	Payment by cash and coupon	28 trips/minute	11 trips/minute/lane	7
	Payment by ETC	4 trips/minute	15 trips/minute/lane	2
case 3	Payment by mixed	31 trip/minute	11 trips/minute/lane	9

In the case of this paper has used the queuing theory to analyzed by M/M/s system for consider the system of service at toll plaza in any type of planning form, if they change the behavior of process toll collection that how affect to the service system. In the M/M/s system has assume the inter arrival time of customer is independent and the character of arrival is exponential distribution (some paper has call in Poisson distribution) (Hiller and Lieberman, 2005) and in every time range of service time has independent respectively. The distribution of service time is character in exponential distribution, the service station has more than one, the definition, notation and symbol of equation has follow as table 1 for analysis the queuing system.

Table 1 definition, notation and symbol for analysis the queuing theory system

symbol	meaning
λ	Mean total arrival rate of vehicles at toll plaza, i.e. the number of arrivals per unit time
μ	Mean service rate for a type i lane, i.e., the number of service completions per unit time
P_n	The probability of customer n unit in the system in any time (from t = 0)
L_q	Mean number of vehicles waiting in the queue at a type i lane
L	average number of customers present in the queuing system at toll plaza
W_q	Mean waiting time of a vehicle in the queue at a type i lane
W	Mean total waiting time in the queue for all arrivals at the toll plaza
ρ	Mean percentage of drivers using at toll plaza

This paper has used the simulation with arena version 10 to analysis in order to find the serviceability of toll plaza in future compare with change the process of payments at the toll booths will affect the advantages or disadvantages. In previous section has analysis the toll plaza by mathematical model. But in the actually situation is so complicate from the factors and the large volume of data that impact to time for analysis has been so long.

The comparison of model has consider to the number of toll booths has used in specific type of payments but not consider in deep to the location of toll booths because the data of many factors not enough.

5. The assumption of the simulation at toll plaza

- The simulation model has used the arrival data at the Dao-khanong toll plaza with case study. From the data has separate from month to daily with 45,129 trips per day when the data has transpose to the time arrival is 1.9 second per trips that is exponential distribution (EXP) in figure 7.

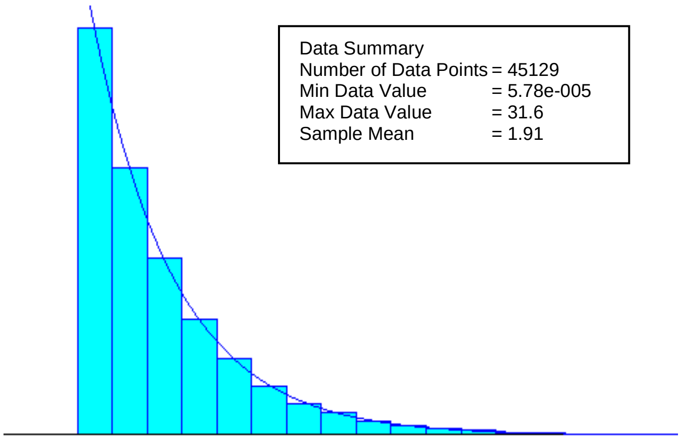

Figure 7: the interarrival time distribution is Exponential Distribution

- The service time has collect from the data at service station is the estimate time under observation and estimate in table 2 as:

Table 2: The definition time process of toll plaza using in simulation

Process Type	**Time process**
Payment by cash	Triangular (6, 8, 12)
Payment by cash without changes	Triangular (3, 5, 8)
Payment by coupon	Triangular (3, 4, 6)
Payment by ETC	Triangular (2, 4, 5)

- The number of toll collection booths has total 9 lane, assume the character of the same type of the toll collection system has stand in near in order to easy to analyzed and this paper has decide the character.

6. The result from analyzed the service process in 3 characters of payment

6.1 Result from analyzed by queuing theory

From the analysis in 3 types of characteristics service has to compare show in table 3 has find the service type by mixed the payment collection service in type 3 has reduce the waiting time, number of customer waiting and used the minimum service time in system with compare between 1, 2 and 3. That result is the decision support at the beginning, the benefit from model is flexible for service in all payment at all lanes which help the service level to quick and decrease the traffic jam at the front of toll collection plaza as well.

Table 3: the comparison in case of service level in any type of payment in each lane

case 1		ρ	P(0)	Lq	L	Wq	W
	Payment by cash	0.666667	0.11111	8	10	0.5333	8.1333
	Payment by cash without changes	0.416667	0.41178	0.425	1.2583	0.12583	0.2092
	Payment by coupon	0.1	0.8181	0.00202	0.20202	0.06734	0.134
	Payment by ETC	0.133333	0.7647	0.004826	0.271526	0.0012	0.06786
case 2		ρ	**P(0)**	**Lq**	**L**	**Wq**	**W**
	Payment by cash and coupon	0.363636	0.07797	0.0097394	2.5597	0.0003478	0.0913
	Payment by ETC	0.133333	0.7647	0.004826	0.271526	0.0012	0.06786
case 3		ρ	**P(0)**	**Lq**	**L**	**Wq**	**W**
	Payment by mixed	0.301615	0.0665	0.0134	2.7234	0.00043	0.088

6.2 Result from analyzed by simulation

From the comparison of 3 systems by consider from the result of 3 cases thus:

Table 4: The waiting time before serve service (Unit: Minutes)

Case Study / Measurement indicators	case 1	case 2	case 3
The average waiting time of customers pay by cash	1.8589	0.0685	0.0088
The average waiting time of customers pay by cash without changes	0.8061	0.0685	0.0090
The average waiting time of customers pay by coupon	0.0288	0.0687	0.0087
The average waiting time of customers pay by electronic pass	0.0249	0.0263	0.0091

From table 4 has consider about the customer number in queue in 3 case to comparison has result of the customer service capacity in queue payment by cash has decrease in case 3 when compare with the case 1 and 2. From the result in case 1 (see table 5) about each type of payment to service in each lane has incur the queue number to 0.45 trips has waiting, but when we change the pattern of payment that result of case 2 and 3 has reduce to 6 trips to waiting in queue. That show the effectiveness of service level when we change the pattern of payment and the service lanes has help to reduce the waiting queue number in system. And represent to the change of pattern of payment in the toll system in mix service payment has great than the service payment by each type in each lane, the advantage of mixed payment in one tollbooth has reduce the traffic jams, improve the efficiency of service and reduce the incidence in the near area of toll plaza also.

Table 5: the number customer has waiting in queue

Case study / Measurement indicators	case 1	case 2	case 3
The average number of customers pay by cash	0.4592	0.0169	0.0021
The average number of customers pay by cash without changes	0.1402	0.0119	0.0015
The average number of customers pay by coupon	0.0014	0.0032	0.0004
The average number of customers pay by electronic pass	0.0014	0.0015	0.0005

Table 6: The total time of customer has used in the system in each payment process (Unit: Minutes)

Case study / Measurement indicators	case 1	case 2	case 3
The total time of customers pay by cash has used	9.8593	8.7349	8.6752
The total time of customers pay by cash without changes has used	6.1389	5.4022	5.3426
The total time of customers pay by coupon has used	4.3614	4.0697	4.0097
The total time of customers pay by electronic pass has used	3.6917	3.6929	2.0028

Table 7: The resource utilization has work in the system in each payment process

<table>
<tr><th>Case study / Measurement indicators</th><th>case 1</th><th>case 2</th><th>case 3</th></tr>
<tr><td>The resource utilization of payment by cash</td><td>65.86%</td><td rowspan="3">44.18%</td><td rowspan="4">38.56%</td></tr>
<tr><td>The resource utilization of payment by cash without changes</td><td>46.36%</td></tr>
<tr><td>The resource utilization of payment by coupon</td><td>10.25%</td></tr>
<tr><td>The resource utilization of payment by electronic pass</td><td>10.61%</td><td>10.61%</td></tr>
</table>

From the table 6 and 7 have considered the total time of customers has used in system and resource utilization the toll booths. From the table 6 has a result in case 1 used service time more than case 2 and 3 respectively. The case of total time pay by cash has more than another because the number of traffic is too much, but result is different in the 3 case because the number of service station in many type of payments and the utilization in each case (see table 7).

7. Conclusion and Suggestion

From the three groups of characters of collection payment at toll plaza has the capacity of arrival and the service time at the same in all type but has consider in the pattern of service to open at the tollbooths with the specific lane in specific type or the mixed service type in all service lanes. From the analysis has help the decision supports to planning the pattern of collection service in tollbooths and to manage the traffic in the front of toll plaza in all case in order to improve the efficiency of traveling to the drivers has serve service, and reduce the waste in any case just be incur to affect the service levels.

From the information would be conclude with the pattern of collection service payment in mixed all type of payment in all lanes from the comparing result with the efficiencies of service level, the waiting time, and the queue number has in minimal, that including to the responsiveness for the service in any type has been quickly, helps the queue has well-disciplinary and smoothly, and can support when the future capacity will be increase than situation. If the expressway have used the all tollbooths by the electronic toll collection, that will help the transportation in the Bangkok and circumference to be quickly.

8. Suggestion

The pattern in the 3 characters to analysis has considered under the data detail in capacity of service level in the day, the service time to serve customers at the toll plaza. The result can help to decision support in the beginning of planning to improve service levels. But the many factors has not consider in this case because the data cannot to collect in situation. In the actually of work has many factors of environment to affect the service level i.e. the type of car and etc. if we know this factors must help to forecast the correct result by simulation for help to decision the planning with correctly and optimized for the service level. Especially, the financial data has to applied in simulation must help to planning the strategies to manage the toll plaza and reduce the risk investment management for operation also.

9. Acknowledge

I am special note of thank to the expressway authority of Thailand to give many data for study in this case. And also grateful for the statistics office and the collection data office of the expressway authority of Thailand has help and advise the way to used data in this case.

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