A Case Study of Using Integer Programming for the Mobile Health Checkup Scheduling

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Abstract

Health checkup is an examination of body's function in order to prevent illness and detect the disease that has no apparent symptoms. According to the social welfare and health maintaining trend, the health checkup becomes familiar service. As the health checkup obtained at the hospital is the general process, many hospitals and clinical companies offer a new service called mobile health checkup service to provide their customers' convenience. In this study, we considered the application of the traveling salesman problem to the mobile health checkup scheduling where the firm needs to provide checkup services to the customers at visiting points distributed around the country. The aim of this study is to apply the mathematical model to the real problem and compare the results with the company's scheduling. We formulated an integer programming and used LINDO 6.01 to solve the numerical study. The results showed that the percentage of saving distance for the middle and the East, the North, and the Northeast are 69.51%, 44.43%, and 34.79%, respectively. The total saving distance is 5,139.77 kilometers or approximately 50.31% compared to the current company's plan.

Keywords: mobile health checkup scheduling; traveling salesman problem; integer programming;

1. Introduction

In business world, the more profit they gain, the more chance they survive. As a result many companies try to earn profit as much as they can. There are two ways to gain more profit; to reduce the cost or to set the product price at the proper level. Although, pricing the company's product can increase its profit, it is not much practical. In particular, a monopoly is rarely exist in most of the markets. In the competitive market, it is usual that people choose to buy the product at the lower price. Therefore, in this case, pricing strategy is no longer a proper way and reducing the cost is the better choice.

Health checkup service is an examination of body's function. In general, health checkup package includes blood pressure screening, blood test, urine test, eye exam and chest x-ray. There are many types of health checkup depending on the risk in career, age, and gender. For example, the health screening for women age 18-39 adds cholesterol screening, immunizations, breast exam and pelvic exam. The purposes of health checkup are to prevent illness, identify risk factors for common chronic diseases and detect disease that has no apparent symptoms. Moreover, health checkup is a way for doctor to counsel people to promote their health behaviors. The earlier the disease is found, the more chances for treatment and cure. Consequently, they can save the medical treatment expense as well as save their lives. Thus the health checkup becomes essential in providing health surveillance.

Although, the health checkup service is provided at most hospitals, many hospitals and clinical companies offer a mobile health checkup service to provide their customers' convenience. The mobile health checkup takes place outside the hospitals while people can obtain the services like in the hospitals. Although the mobile health checkup service can increase the number of customers or the company's incomes, the mobile health checkup also increases the cost in transportation. According to the study in mobile health unit of National Rural Health Mission, Department of Health & Family Welfare, Government of Orissa, India [1], the transportation cost is approximately 38.24% of the overall cost in KBK district.

It is explicit that the transportation cost shares a high proportion of the total cost in the mobile health checkup service. The reduction of transportation cost can help to decrease the total cost of this service. Assigning the vehicles to the appropriate places with the suitable routes is an efficient and well-known method to reduce the transportation cost. This method is known as the vehicle scheduling. The vehicle scheduling is not only save the transportation cost but also save the traveling time and help the company's processes run smoothly.

This paper is focused on the vehicle scheduling for the mobile health checkup service which the vehicles have to leave the company, visit every customer places to provide services and finally return to the company. The objective of this study is to find the optimal route planning that cover all visiting places and save the traveling distance as much as possible. The whole service area was separated into a smaller region. The mathematical model was formulated to describe this mobile health checkup scheduling problem. After that the optimal route which provides the shortest traveling distance in each region was found by LINDO 6.01. The results from the package were compared with the company's plan.

2. Basic Theory

2.1. The Assignment Problem

The assignment problem is a problem which assigns each agent to the task. Any agent can be assign to perform any task. The associated cost of assigning agent i to perform task j is a_{ij}. It is required to perform all tasks by assigning exactly one agent to each task in such a way that the total cost of the assignment is minimized. The mathematical model of the assignment problem with m agents and n tasks can be written as the following.

$$\text{Minimize} \quad \sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij} y_{ij} \tag{1}$$

$$\text{subject to} \quad \sum_{i=1}^{m} y_{ij} = 1 \qquad \text{for } j = 1,2,...,n \tag{2}$$

$$\sum_{i=1}^{n} y_{ij} = 1 \qquad \text{for } i = 1,2,...,m \tag{3}$$

$$y_{ij} \quad \text{is binary} \qquad \forall i, j \tag{4}$$

where $y_{ij} = \begin{cases} 1 & \text{if agent } i \text{ performs task } j, \\ 0 & \text{otherwise} \end{cases}$ for $i = 1,2,...,m;\ j = 1,2,...,n.$

The assignment problem is widely used to apply in many problems such as the workforce scheduling [2,3,4,5]. Pastor and Olivella [2] studied the weekly work schedules with working time accounts. In this problem, they decided the capacity of employees that meets the level of services and minimized the shortages. In the healthcare service, the nurse scheduling is one of the application of the assignment problems which each nurse is assigned to the task in a specific period [4]. A heuristic approach was used to improve the nurse training schedules by Nooriafshar [5]. Yan et. al [6] applied this assignment model to in the airport gate assignments. The university course timetable was studied by Dimopoulou and Miliotis [7] in order to develop the automated timetable construction system by assigning each course to the specific time period.

2.2. The Traveling Salesman Problem

The traveling salesman problem is a special case of the assignment problem. A salesman has to visit a prescribed set of cities and return to the original location in such a way that the total traveling distance is minimized and each city is visited exactly once. The traveling salesman problem is also called a combinatorial mathematical programming.

Let b_{ij} be the distance from City i to City j,

$|S|$ be the number of elements in set S,

$z_{ij} = \begin{cases} 1 & \text{if the route includes a path from City } i \text{ to City } j, \\ 0 & \text{otherwise} \end{cases}$ for $i = 1, 2, ..., n;\ j = 1, 2, ..., n.$

Then the traveling salesman problem can be formulated as

$$\text{Minimize} \quad \sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij} z_{ij} \tag{5}$$

$$\text{subject to} \quad \sum_{i=1}^{n} z_{ij} = 1 \qquad \text{for } j = 1,2,...,n \tag{6}$$

$$\sum_{i=1}^{n} z_{ij} = 1 \qquad \text{for } i = 1,2,...,n \tag{7}$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n} z_{ij} \leq |S|-1 \qquad \forall S \subset \{1,2,...,n\}, S \neq \phi, \forall i,j \tag{8}$$

$$z_{ij} \quad \text{is binary} \qquad \forall i,j\,. \tag{9}$$

Observe that the mathematical model of the traveling salesman problem is similar to the assignment problem in Section 2.1 except Equation (8). This constraint guarantees that there is no subtour because only one tour visiting all cities is desired.

Traveling salesman problem is classified as a combinatorial optimization problem because it has a large number of feasible solutions. Many methods and strategies were developed to find the optimal solution in the traveling salesman problem. Since the traveling salesman problem is an integer programming, the methods for solving an integer programming called branch-and-bound and branch-and cut can be applied. Hernàndez-Pèrez et. al [8] used the branch-and-cut method to solve the traveling salesman problem with pickup and delivery. Cordeau et. al [9] not only used the same method to solve the traveling salesman problem with pickup and delivery but also added the first-in-first-out policy.

The traveling salesman problem is an NP-hard problem. Thus, a number of researchers chose a heuristic method that provides a good and fast-finding solution to solve this problem. A heuristic algorithm is a popular method used to solve the optimization problem. The idea of this method is to find a good solution by improving the initial feasible solution. Renaud et. al [10] proposed a two-phase heuristic algorithm to solve the traveling salesman problem with pickup and delivery. A special heuristic method called a tabu search was applied in this problem by Gendreau et. al [11] to find some near-optimal solutions.

3. Problem Description

In this study, we applied the traveling salesman problem to the real health checkup problem. It can be determined as the vehicle scheduling problem of a company. This company obtained the annual health checkup task which contains the customers throughout the country. The health checkup vehicles of the company need to visit each customer in order to provide services. The company separated the area of Thailand into four regions geographically. Region A is the area of the middle and the East of Thailand. Region B is the North of Thailand, Region C contains the Northeast of Thailand and Region D covers the South of Thailand. In this study, we considered a numerical study for Regions A, B, and C only. However, the mathematical formulation for Region D is very similar the others. The visiting places in each region are shown in Table 1. The cluster of visiting places that the company has to visit in each region is demonstrated in Figures 1-3.

Table 1: The visiting places in each region

Region	Area	The number of visiting places
A	The middle and the East of Thailand	22
B	The North of Thailand	18
C	The Northeastern of Thailand	19

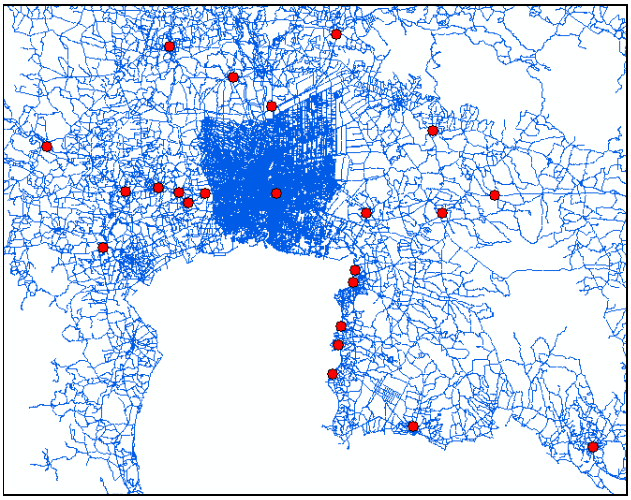

Figure 1: The cluster of places in Region A

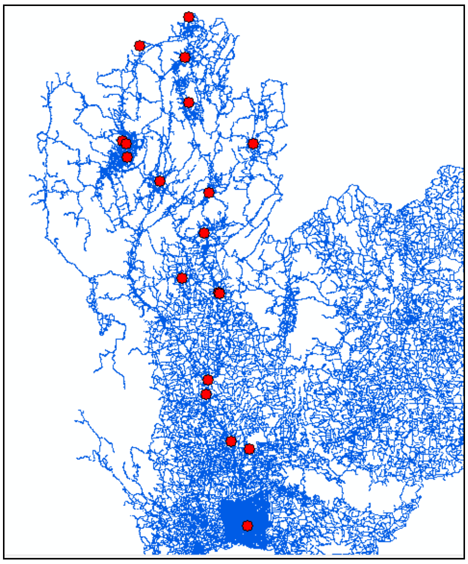

Figure 2: The cluster of places in Region B

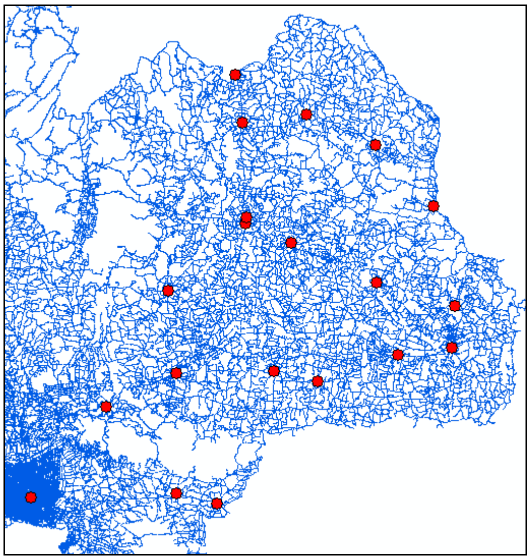

Figure 3: The cluster of places in Region C

4. Research Methods

In this problem, we needed to find the most economical health checkup route. We assumed that the transportation cost is proportional to the traveling distance. Then the shortest distance route in each region was determined. To identify the order of visiting places we first determined the distance between each place and formulated the mathematical model. Then the distance matrix was applied to the objective function of the mathematical model. Thus, this section can be separated into two parts. The first part demonstrates how to find the distance matrix and the later part shows the mathematical formulation of this problem.

<u>4.1. Distance Matrix</u>

4.1.1. Find the coordinate (latitude and longitude) of the company and all visiting places. Google maps is used to identify them.

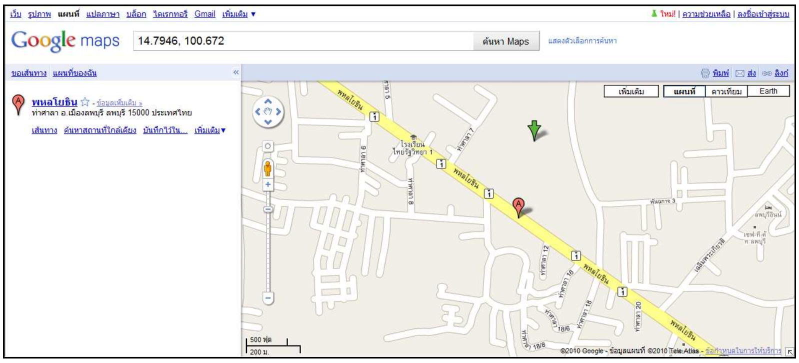

Figure 4: Example of the coordinate in Google maps

4.1.2. Fix the point of the company and all visiting places in each region by using Google Earth.

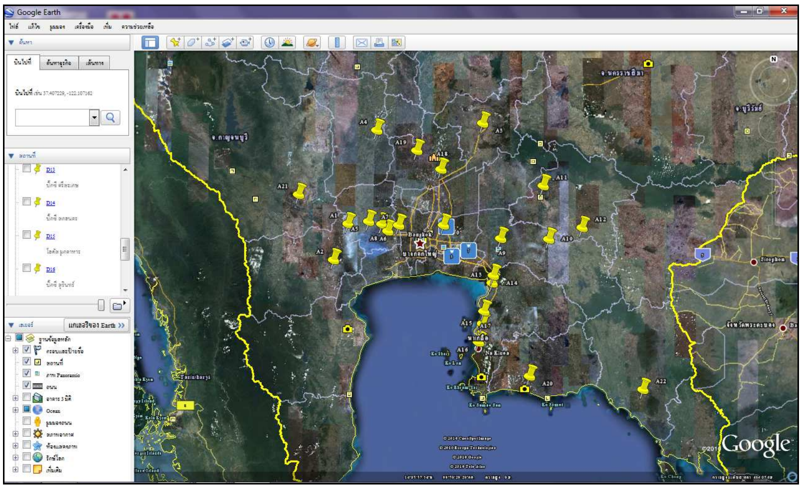

Figure 5: Example of the location point in Google Earth

4.1.3. Use DNR Garmin 5.4.1 to transform the file obtained from step 2. Now the transformed file is ready to use in ArcMap 9.3.1.

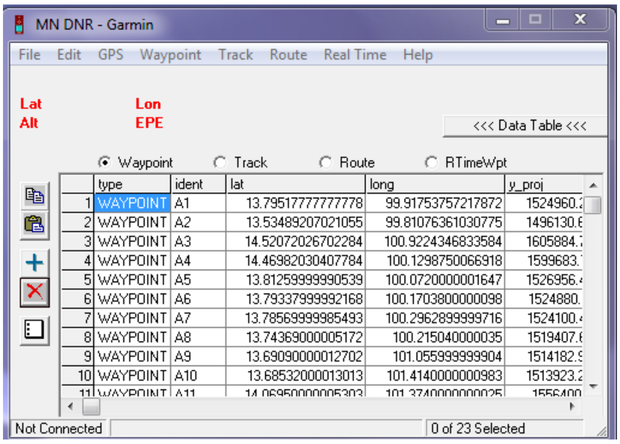

Figure 6: Example of the file transformation in DNR Garmin

4.1.4. Find the distance between each point by using ArcMap 9.3.1.

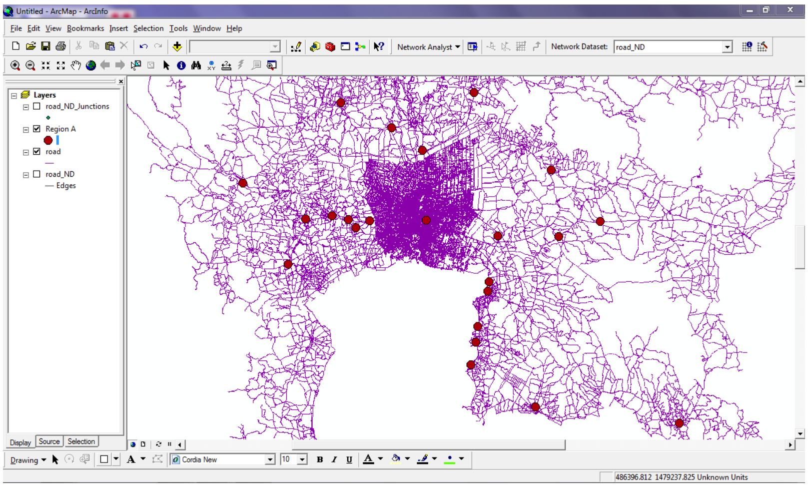

Figure 7: Example of the distance matrix computation in ArcMap

4.2. Mathematical Formulation

The restrictions of the health checkup planning are as followings.

1) The company has 2 mobile health checkup vehicles.
2) Every vehicle must leave the company to all visiting places in the region and return to the company after finish the mobile health checkup service.
3) Each visiting place must be visited only once.

To formulate the mathematical model for this problem, we used the following notations.

Let c_{ij} be the traveling distance from visiting place i to visiting place j obtained from Section 4.1,

N be the number of visiting places in the region,

$$x_{ij} = \begin{cases} 1 & \text{if the vehicle goes from visiting place } i \text{ to visiting place } j, \\ 0 & \text{otherwise} \end{cases}$$

Note that the visiting place 0 is the company. Then the integer programming for the health checkup problem was

$$\text{Minimize} \quad \sum_{i=0}^{N}\sum_{j=0}^{N} c_{ij} x_{ij} \tag{10}$$

$$\text{subject to} \quad \sum_{i=0}^{N} x_{ij} = 1 \qquad \text{for } j = 0,1,2,\ldots,N \tag{11}$$

$$\sum_{i=1}^{N} x_{ij} = 1 \qquad \text{for } i = 0,1,2,\ldots,N \tag{12}$$

$$u_i - u_j + N x_{ij} \le N - 1 \qquad \text{for } i = 1,2,\ldots,N,\ j = 1,2,\ldots,N,\ i \ne j \tag{13}$$

$$x_{ij} \quad \text{is binary} \qquad \forall i, j \tag{14}$$

$$u_i \ge 0 \qquad \text{for } i = 1,2,\ldots,N \tag{15}$$

The objective function (10) aimed to minimize the total distance of the tour. Equations (11) and (12) guaranteed that for every visiting place there was exactly one vehicle reach and leave it. These equations followed from the assumption that every visiting place must be visited exactly once. Equation (13) insisted that there was only one tour serving all visiting places. Equations (14) and (15) assured the values of all x_{ij} and u_i. This integer programming was the model for the traveling salesman problem.

After obtaining the mathematical formulation, the distance matrix obtained in Section 4.1 was used as the coefficients of the objective function (1). For each region, we solved the integer programming by using LINDO 6.01 run on a 2.4 GHz Intel® Pentium® Dual CPU, 2.00 GB RAM and Window Vista™.

5. Results and Analysis

The optimal route of each region obtained from LINDO 6.01 is shown in Figures 8-10. In Figure 8, the optimal route of Region A is complicated. This figure indicates that part of the route is not a cycle. This means that the company does not need to travel on the same road. As it can be seen that the Region A covers the middle and the East of Thailand, since these areas are close to Bangkok there are several main routes from one place to the others. Hence there are many alternative routes for traveling between two places. This creates the small cycles for traveling.

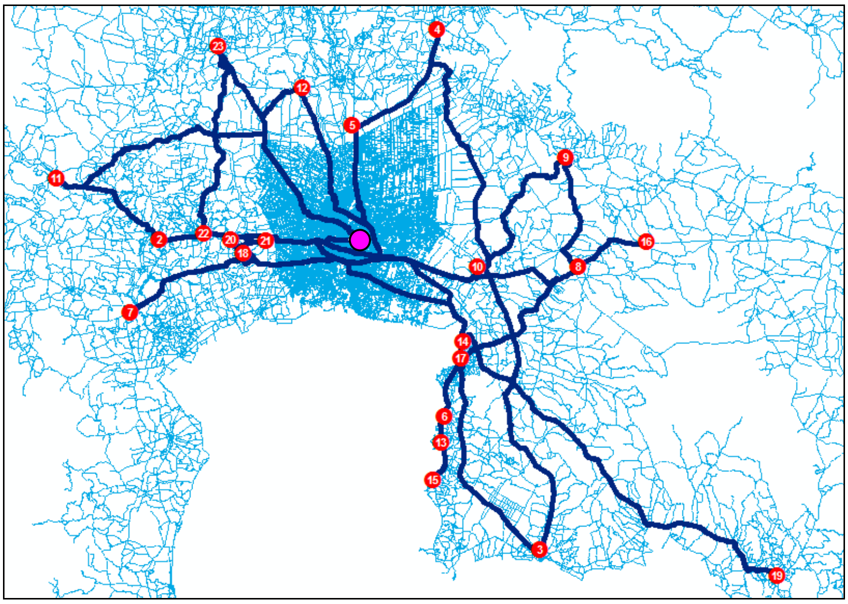

Figure 8: The optimal route for Region A

Unlike Region A, the optimal routes of Region B (the North) and Region C (the Northeast) shown in Figures 9 and 10 obviously have less cycles. In fact, each of the regions contains only one main cycle, due to the geography of the area and limited highways linked between provinces. In Figure 9, the optimal route of Region B takes the same partial route in traveling on the way back. Similarly, the optimal route of Region C in Figure 10 is precisely a cycle because there are two main highways for linking the provinces in the north and the south of the Northeast.

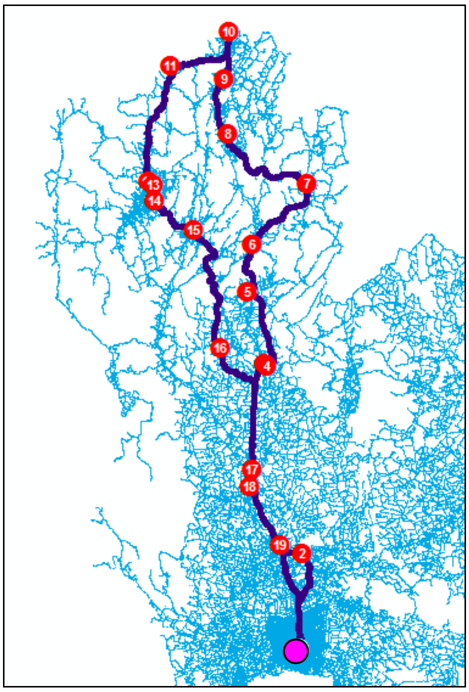

Figure 9: The optimal route for Region B

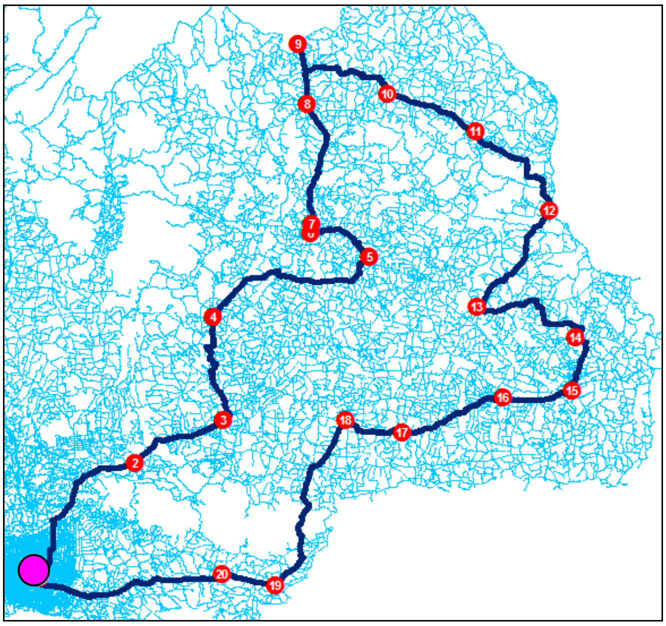

Figure 10: The optimal route for Region C

The total traveling distance of the optimal route and the company's planning in each region was shown in Table 2. The optimal solution is 1,092.45 kilometers while the company's plan for Region A is 3,582.91 kilometers. For Region B, the company's plan is 3,546.14 kilometers while the optimal solution is 1,970.60 kilometers. The plan of the company for Region C is 3,086.32 kilometers and the optimal solution shows the result of 2,012.55 kilometers. As indicated in Table 2, using the mathematical model to find the optimal solution saves the company's traveling distance for all regions. The optimal routes of Regions A, B and C reduce the traveling distance 2,490.46 kilometers, 1,575.54 kilometers and 1,073.77 kilometers, respectively comparing to the company's plan. The total saving distance is 5,139.77 kilometers. Saving percentage for Region A is 69.51%, Region B is 44.43% and Region C is 34.79%. For the total traveling distance using the mathematical model can save total company's traveling distance approximately 50.31%.

Table 2: The total traveling distance in each region

Region	Total distance (km.)			
	Optimal solution	Company's planning	Saving distance	Saving Percentage
Region A	1,092.45	3,582.91	2,490.46	69.51%
Region B	1,970.60	3,546.14	1,575.54	44.43%
Region C	2,012.55	3,086.32	1,073.77	34.79%
Total	5,075.60	10,215.37	5,139.77	50.31%

6. Conclusions

We studied the real vehicle scheduling problem for the mobile health checkup service of a company which has to provide services to the customers throughout the country. The company's vehicle leaves the company, visits every customer and returns to the company. Without loss of generality, most of the traveling cost is proportional to the traveling distance. Thus, the objective of this work was to find the minimum distance route for the health checkup in each region. We formulated the integer programming for this problem and implemented to the real problem. The company's data is considered to find the distance matrix of the visiting places and then LINDO 6.01 is used to solve the mathematical model. The visited places were seperated into 3 regions. The optimal route in each region was obtained and compared with the present plan of the company. The results showed that using the integer programming to find the route in each region can save the company's traveling distance. The percentage of saving distance for the middle and the East, the North, and the Northeast are 69.51%, 44.43%, and 34.79%, respectively. The total saving distance is 5,139.77 kilometers or approximately 50.31% compared to the current company's plan.

In this study, we considered the case of only one vehicle was used in each region. Nevertheless, in many problems more than one vehicle can be need for some locations depends on the number of customers in each location. For the future work, we will extend the mathematical model to the multi-vehicle case. Moreover, it would be interested to consider additional timeline constraints in the vehicle scheduling.

7. References

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